Volatility correlations between short and long-term interest rates in South Africa

Abstract

Volatility correlations of short- and long-term interest rates are investigated using the diagonal VECH and BEKK models. The data covers a period of 10 years at weekly frequencies obtained from the Reserve bank of South Africa website. The two interest rate series are found to be different from that of a normal distribution. The diagonal VECH model shows a significant relation between the short- and long-term interest rates as the past shocks and conditional volatility from one interest rate affect the other interest rate's current volatility. In contrast, the diagonal BEKK, which can also measure the volatility spill-over effect, rejects any relation between the short- and long-term interest rates.

Introduction

The relationship between interest rates of differing maturities have been studied extensively in the past. The study of the behaviour of interest rates and their co-movements are important because interest rates are important variables in the pricing and hedging of financial instruments. Cargill and Meyer (1972) state that for any evaluation of open market operations strategies, the relationship between short- and long-term interest rates is of critical importance. The decision-making of institutions such as companies, governments, and households are affected by the behaviour of interest rates (Kobayashi, 2004). Mankiw and Summers (1984) state that the term structure of interest rates is important to economic agents because of its use in the pricing of securities such as bonds and for the evaluation of macroeconomic policies.

This study will examine the variance and covariance of short- and long-term interest rates in South Africa. Interest rate volatility is important to several economic agents for different purposes such as asset allocation and portfolio selection decisions by portfolio managers, measurement of market risk by risk managers, pricing and hedging of derivative securities by traders, and regulators to supervise and monitor financial institutions behaviour (Ferreira, 2005). The purpose of this study is to model the variance and covariance of short- and long-term interest rates in South Africa using the multivariate variance model-based approach. Ferreira (2005) states that univariate time-varying models have been used extensively in the literature, whereas works on multivariate approaches are still lacking.

Literature review

Several authors have studied the relationship between interest rates of differing maturities using different approaches. Iyer (2000) studies the relationship between short-term and forward interest rates using the structural time-series method. The forward interest rate used in the study is the interest rate implied by the difference between the current yield on short and long period bills. The data spans from December 1946 to January 1991. It is found that forward rates have a minor, if any, influence in predicting variations in the short rate (Iyer, 2000).

Kobayashi (2004) shows that the negative relation between short- and long-term interest rates can be as a result of simultaneous occurrences of two economic shocks using various new Keynessian models. The new Keynessian models show that long-term interest rates react differently to changes in short-term interest rates depending on the direction of economic shocks (i.e. demand and cost-push shocks).

Iyke (2015) investigates the dynamic correlations between short-term and long-term interest rates using nonlinear cointegration tests. The study uses the SARB policy rate as proxy for the short rate and the yield on long-term government bonds in South Africa as data that spans from January 1957 to February 2015. The results of the study by Iyke (2015) show that there is a causal flow from the long-term yields to the short-term yields.

The variance-covariance process of weekly short-term spot interest rates in France and Germany from 1981 to 1997 is studied by Ferreira (2005). Using multivariate GARCH models, Ferreira (2005) finds that the variance of the short rate in France is moved by its own news and German news, whereas the variance of the short rate in Germany only responds to its own news and is unaffected by news in France. Additionally, it is found that the French and German interest rates have a higher covariance following a negative shock to German interest rates and smaller covariance after a positive shock to interest rates in France.

Data and Methodology

This study uses 91 day Treasury bills and the current yield on the R186 bond as short-term and long-term rates obtained from the Reserve Bank of South Africa, respectively. The data span the period from February 2009 to February 2019 at weekly frequencies. The two interest rates series are transformed to log-normal interest rate changes. Interest rates are usually considered to be integrated of order one I(1) (Iyke, 2015; Ferreira, 2005).

Multivariate volatility models

Taking into account $\{y_t\}$ as an n-dimensional vector stochastic process. The process $\{y_t\}$ can be written as follows:

$$y_t = \mu_{t(\theta)} + \epsilon_t \tag{1}$$

Where $\mu_t(\boldsymbol{\theta})$ is the conditional mean vector and

$$\epsilon_t = H_t^{\frac{1}{2}}(\boldsymbol{\theta})\boldsymbol{\zeta_t} \tag{2}$$

Where $H_t^{\frac{1}{2}}(\boldsymbol{\theta})$ is an *N* x *N* positive definite matrix. Additionally, $\boldsymbol{\zeta_t}$ is assumed to be the *N* x 1 random vector with the following two second moments:

$$\boldsymbol{E}(\zeta_{\boldsymbol{t}}) = \mathbf{0}$$

$$Var(\zeta_{\boldsymbol{t}}) = \boldsymbol{I_N} \tag{3}$$

Where $\boldsymbol{I_N}$ is the identity matrix of order *N*.

Dropping the $\boldsymbol{\theta}$ (for convenience), $H_t^{\frac{1}{2}}$ can be easily defined by calculating the conditional variance matrix of y_t:

$$\begin{aligned} Var(y_t|I_{t-1}) = Var_{t-1}(y_t) &= Var_{t-1}(\epsilon_t) \\ &= H_t^{\frac{1}{2}} Var_{t-1}(\zeta_{\boldsymbol{t}}) \left(H_t^{\frac{1}{2}}\right)' \\ &= H_t \end{aligned} \tag{4}$$

Thus $H_t^{\frac{1}{2}}$ is any *N* x *N* positive definite matrix such that H_t is the conditional variance matrix of y_t. H_t is thus the conditional variance that is of interest in multivariate GARCH modelling.

The diagonal VECH model

Bollerslev, Engle & Wooldridge (1988) proposed a general formulation of H_t. Each element of H_t is a linear function of the lagged squared errors and cross products of errors and lagged values of the elements of H_t in the general VEC representation (Bauwens, Laurent & Rombouts, 2006; Scherrer & Ribarits, 2007).

The VEC model proposed by Bollerslev et al. (1988) is defined as:

$$h_t = c + A\eta_t + Gh_{t-1} \quad (5)$$

Where

$$h_t = vech(H_t) \quad (6)$$

$$\eta_t = vech(\varepsilon_t \varepsilon_t') \quad (7)$$

Where c is a $(N(N+1)/2)$-dimensional vector and A and G are $(N+1)N/2$ square matrices. The VEC model has several weaknesses; notably, the number of parameters are very large: $N(N+1)(N(N+1)+1)/2$ which is large for even a small N. In the instance that $N = 3$ there are 78 parameters in the model. Bollerslev et al. (1988) suggest the diagonal VEC model to overcome this problem by imposing some simplifying assumptions. In the diagonal VEC, the A and the G are assumed to be diagonal, in which each element h_{ijt} depend only on its own lag and on the previous value of $\varepsilon_{it}\varepsilon_{jt}$. Thus, the number of parameters $N(N+5)/2$, for $N = 3$ are reduced to 12 with the imposed restriction.

The diagonal VEC model can then be defined as:

$$H_{t=} C^{\circ} + A^{\circ} \odot (\varepsilon_{t-1}\varepsilon_{t-1}') + G^{\circ} \odot H_{t-1} \quad (8)$$

Bauwens et al. (2006) states that H_t is positive definite for all t provided that the initial matrix $((H_0)$, C°, A° and G° are positive definite.

Diagonal BEKK

The BEKK(*p*,*q*,*K*) model proposed by Baba, Engle, Kraft & Kroner (1991) can be defined as:

$$H_t = CC' + \sum_{i=1}^{q} \sum_{k=1}^{K} A_{i,k}' \varepsilon_{t-1}\varepsilon_{t-1}' A_{i,k} + \sum_{j=1}^{p} \sum_{k=1}^{K} G_{q+j,k}' H_{t-j} G_{q+j,k} \quad (9)$$

Where C is a nonsingular $n \times n$ matrix, $A_{i,k}$ and $G_{q+j,k}$ are $n \times n$ matrices. The BEKK model is a special case of the VEC model. The number of parameters in the BEKK model is $N(5N+1)/2$. Bauwens et al. (2006) state that to reduce the number of parameters and consequently the generality, the diagonal BEKK can be imposed. That is, A_k and G_k are diagonal matrices. The diagonal BEKK is also the diagonal VEC model but is less general, though as opposed to the diagonal VEC model, is guaranteed to be positive definite.

Empirical Results

Descriptive statistics

The data are transformed to log returns in order to analyse them statistically. Table 1 below gives the descriptive statistics of the R186 long term bond and the 91-day T-bill log returns.

Table 1: Descriptive statistics of R186 and 91-day T-bill

	LOG_R186	LOG_TBILL
Mean	0.000122	-0.00041
Median	-0.00112	0
Maximum	0.183091	0.076174
Minimum	-0.1075	-0.09261
Std. Dev.	0.020879	0.01168
Skewness	1.193062	-1.45933
Kurtosis	15.04323	24.43274
Jarque-Bera	3303.564	10254.39
Probability	0	0
ADF t-Statistics (Prob.)	-25.4206 (**p<0.01**)	-21.0966 (**p<0.01**)
Observations	526	526

As can be seen from Table 1, the log of R186 and 91-day T-bill have means that are close to zero. The R186 has the highest maximum and lowest minimum of 0.1831 and -0.1075, respectively. The R186 also has the highest standard deviation of 0.0209 and the 91-day T-bill with a standard deviation of 0.0117. The third and fourth moments of the R186 and 91-day t-bill distributions are different from that of a normal distribution. The R186 and 91-day T-bill have skewness of 1.1931 and -1.4593, with the 91-day T-bill showing negative skewness as commonly found in financial time series data. Both instruments show excess kurtosis of 12.0432 and 21.4327 for R186 and 91-day T-bill, respectively. The normality assumptions is rejected for both the R186 and the 91-day T-bill as shown by their respective Jarque-Bera statistics of 3303.564 and 10254.39 and probabilities that are p<0.01. the time series of the two instruments reject the unit root hypothesis as tested by the Augmented Dickey-Fuller test statistic.

Multivariate GARCH Model estimation results

Table 2 below reports the results of estimating the diagonal VECH and BEKK models.

Table 2: Diagonal VECH and BEKK estimation

Transformed variance equations		
	Diagonal VECH	**Diagonal BEKK**
M(1,1)	0.0002 (8.7252)***	0.0001 (1.4580)
M(1,2)	**1.26E-05 (4.4970)*** **	1.34E-05 (1.5127)
M(2,2)	3.35E-06 (8.1982)***	4.46E-06 (11.2973)***
A1(1,1)	0.0759 (5.5134)***	0.2252 (2.8501)***
A1(1,2)	**0.0407 (7.3831)*** **	-
A1(2,2)	-0.0002 (-0.0884)	0.0059 (0.1488)
B1(1,1)	0.4653 (9.0265)***	0.8209 (6.2970)***
B1(1,2)	**0.7147 (23.4681)*** **	-
B1(2,2)	0.9708 (288.7913)***	0.9784 (538.8652)***

*Note: *,**, and *** denote 10%, 5%, and 1% significance level. Numbers in parenthesis represent z-statistics. Columns in* ***bold*** *signify statistically significant relationship between the R186 and 91-day T-bills.*

As can be seen from Table 2, the constant terms of the variance equation denoted by M in the diagonal VECH are statistically significant at a 1% confidence level and there appears to be a significant relation between the R186 and the 91-day T-bill. Whereas in the diagonal Bekk the relation is insignificant, with only the 91-day T-bill constant of 4.46E-06 being statistically significant at a 1% confidence level. The A1 represents the ARCH effect. The R186 is shown to be affected by its own past shocks while the 91-day T-bill shows no ARCH effect in its variance according to both the diagonal VECH and BEKK. Moreover, the R186 and the 91-day T-bill have a significant relationship as shown by the A1(1,2) coefficient in the diagonal VECH, implying that a shock from the R186 affects the variance of the 91-day T-bill and vice-versa. The GARCH parameters of the diagonal VECH and BEKK are all statistically significant. The variance of the R186 and the 91-day T-bill are both affected by their own past shocks and conditional variances. Additionally, the diagonal VECH model shows that the variance of the 91-day T-bill is affected by the past shock and conditional variance of the R186 and vice versa. The diagonal VECH model gives an indication that there is a statistically significant relationship between the R186 and the 91-day T-bill. In contrast, the diagonal BEKK rejects the existence of such a relation.

Diagnostic check

Bauwens et al. (2006) state that the portmanteau tests are the most common diagnostics to detect ARCH effects. Additionally, the tests are used to detect misspecification in the conditional variance. Table 3 reports the results of the portmanteau tests for the Multivariate GARCH system residuals.

Table 3: Portmanteau tests for autocorrelations of standardised residuals

	Diagonal VECH	**Diagonal BEKK**
	$\epsilon_t(\sqrt{corr})$	$\epsilon_t(\sqrt{corr})$
Q(12)	118.4831 (0.00)	58.8586 (0.1354)
Q(20)	136.2548 (0.00)	88.9485 (0.2312)
	$\epsilon_t(\sqrt{cov})$	$\epsilon_t(\sqrt{cov})$
Q(12)	69.2524 (0.02)	58.6749 (0.1390)
Q(20)	98.1836 (0.08)	88.8229 (0.2340)
Source: *Researcher's own construct*		
Note: numbers in parenthesis are probabilities. $\sqrt{corr}$ and $\sqrt{cov}$ resemble the square root of correlation and square root of covariance of the standardised residuals.		

The results reported in Table 3 show that the residuals in the Diagonal VECH model are statistically significant at a p<0.01 significance level when the standardised residuals are described using the square root of correlation. It can therefore be said that there is dependence in the standardised residuals resulting from a Diagonal VECH model. In contrast, the Diagonal BEKK model shows no autocorrelation in its standardised residuals in both cases when the residuals are described using the square root of the correlation and the square root of the covariance. The residuals of the Diagonal VECH evidence temporal dependence even when the standardised residuals are described using the square root of the covariance at a

5% and 10% level of significance at 12 and 20 lags, respectively. Thus, the diagonal BEKK model shows that there are no significant autocorrelations in the residuals and no heteroscedasticity problems.

Discussion

The Diagonal BEKK model used in this study is unable to capture any volatility correlations between the short- and long-term interest rates in South Africa. Ferreira (2005) finds that the BEKK model is able to capture volatility spill-overs from the German interest rates to the French interest rates, thus, contrary to this study, there exists a significant relationship between the two interest rates investigated using the BEKK model. Cargill and Meyer (1972) evidence a close relation between short- and long-term interest rates in the U.S. when using a distributed lag model. The results of the authors are in agreement with those of the Diagonal VECH model used in this, but are in contrast to the Diagonal BEKK model. In accord with the Diagonal VECH model but in contrast with the Diagonal BEKK model employed in this study, Iyke (2015) finds evidence of a significant relationship between short- and long-term interest rates in South Africa when applying a cointegration approach to modelling the relationship between interest rates. The results from different researchers using different model approaches suggest a difference in their findings based on different model approaches. Research on multivariate volatility modelling is sparse and thus the variance-covariance of interest rates in the literature is also limited (Ferreira, 2005).

Summary

This study investigated the possible variance-covariance relation between the long-term yield on the R186 and the short-term yield on 91-day treasury bills of the South African interest rate market. The distributions of the time series of the two instruments are shown to be different from that of a normal distribution. Additionally, the log-normal returns of the R186 and the 91-day T-bills reject the presence of a unit root when using the ADF test statistic. The variance-covariance of the short- and long-term interest rates was investigated by employing the diagonal VECH and BEKK models. It was evidenced that there exists a possible relationship between the two interest rates when considering the diagonal VECH model; however, the diagonal BEKK model rejects the existence of any correlation/spill-over between the R186 and the 91-day T-bill.

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