SOUTH AFRICAN FINANCIAL MARKET VOLATILITY IS ROUGH

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Introduction

Volatility, defined as the periodic (annualized) standard deviation of financial asset returns, is a critical factor in financial markets. Consequently, modeling volatility has become a matter of paramount importance. Historically, volatility modelling presents two persistent stylised facts. The first one is the presence of long memory features in volatility. Long memory is exhibited in a stationary process if its covariance function decays slowly. The second stylised fact is the leverage effect which is the existence of the negative correlation between price increments and volatility increments. Recent empirical evidence suggests that most financial markets exhibit the so-called *rough volatility* dynamics or irregular behaviour over time which is modelled via a fractional Brownian motion (fBm) (see Mandelbrot and Ness (1968)) with a Hurst parameter between 0 and 0.5. This empirical observation is credited to the availability of high-frequency datasets and modelling volatility dynamics using rough specifications offers improved pricing, hedging and forecasting performance (see for example Bayer et al (2016), Gatheral et al (2018), Alfeus and Nikitopoulos (2022)).

Rough Fractional Stochastic Volatility (RFSV) models

Rough Fractional Stochastic Volatility models are continuous time stochastic volatility models where the instantaneous volatility process is driven by a fBm with the Hurst parameter smaller than half and emerged in many academic research papers since the seminal paper titled "Volatility is rough" was posted on SSRN in 2014 showing that the log realized volatility time series of major stock indices have the same scaling property as that of a rough fractional Brownian motion has. Gatheral et al (2018) demonstrated that price and option data are more consistent with the Hurst parameter value closer to zero and with a mean reversion parameter for the volatility process kept very small. By virtue of this observation, increments of a fractional Brownian motion are negatively correlated which is a necessary feature for intermittency and anti-persistency and in this case leverage effects are well-taken into account. Gatheral et al (2018) consider the most successful stochastic volatility model originally proposed by Heston (1993). Instead of having the instantaneous volatility process driven by a fractional Brownian motion they convolved the solution of this process with a kernel that explodes around zero. For the Hurst parameter closer to zero, the process results in trajectories that are less regular than for a classical model driven by a Brownian motion.

The RFSV dynamics under a risk-neutral measure is given by:

$$dS_t = rS_t dt + \sqrt{V_t} dW_t^1$$

$$V_t = V_0 + \frac{1}{\Gamma(\alpha)} \kappa \int_0^t (t-s)^{(\alpha-1)} (\vartheta - V_s) ds + \frac{1}{\Gamma(\alpha)} \sigma \int_0^t (t-s)^{(\alpha-1)} \sqrt{V_s} dW_s^2,$$

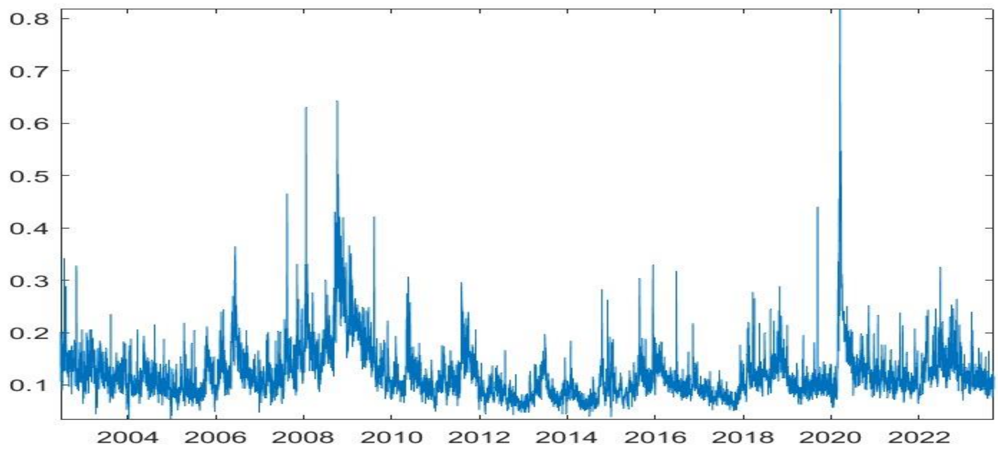
where $\alpha = H + \frac{1}{2}$, $H \in (0, \frac{1}{2})$ is the Hurst parameter, κ is the reversion speed, ϑ is the long-term mean of V_t, σ is the volatility of volatility and W^1 and W^2 are correlated standard Brownian motions, i.e., $< dW_t^1, dW_t^2 >= \rho dt$. Typically, the correlation parameter $\rho \in [-1,1]$ is negative pointing to the fact that a decrease in stock price is correlated with an increase in volatility. It is often required that $2\kappa\vartheta \geq \sigma^2$ to ensure that V_t is always positive. Moreover, in this model setup the kernel $(t-s)^{(\frac{1}{2}-H)}$ is behind the $H-\epsilon$ holder regularity of the volatility for all $\epsilon > 0$. However, this model is neither Markovian nor Semimartingale and this means one cannot use the standard approach of applying bivariate Ito's lemma to derive the fundamental partial differential equations. El Euch and Rosenbaum (2018) consider a microstructure model based on nearly unstable Hawkes processes and designed a suitable rescaling sequence of point processes which converge to the rough Heston model.

The model gives a general interpretation of the volatility dynamics from high-frequency behaviour in the market. Even though fBm seems to be an appropriate tool for modelling volatility (at least from empirical observations and statistical tests) there has been much debate about its applicability for modelling financial derivatives as fBm is not a semimartingle. For example Rogers (1997) showed that fBm could not be used as a price process for a risky security without introducing arbitrage opportunities. As a result, price processes driven by fBm do not satisfy the property of *No Free Lunch With Vanishing Risk* (NFLVR). However, Cheridito (2003) showed that fBm could still be used in a price process under certain conditions: proper restriction of the class of permissible trading strategies is necessary to eliminate arbitrage. Moreover, Jarrow et al (2009) showed that the semimartingale property is not a necessary condition for no-arbitrage and were able to construct a class of processes which are not semimartingales but which remain arbitrage free.

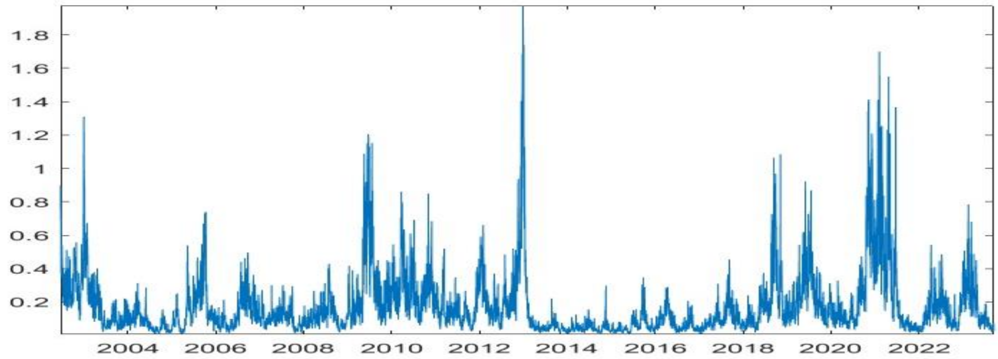
The rough volatility community is well organised, there is even a site that tracks all research on rough volatility since the seminal paper "Volatility is rough".

Rough evidence from South Africa

I consider the high-frequency data (5-min) for the FTSE/JSE All Share Index/ which tracks the performance of all companies listed on the Johannesburg Stock Exchange (JSE). Daily realised variance is then computed from the 5-min intraday data after a rigorous cleaning process especially to remove market microstructural noise. The estimated Hurst parameter is of the order of 0.2, empirically proving roughness in the South African financial market volatility. Below I show the actual realised volatility (top figure) with the simulated rough fractional stochastic volatility model (bottom figure) with $H = 0.2$. The simulated process mimics very well the real data. This gives a strong motivation for rough volatility models in the South African financial market.



Source: Refinitiv DataScope Select : JSE All Share Index realised volatility, time period 24/06/2002 – 15/09/2023



Model: Rough Fractional Stochastic Volatility

Financial implications of RFSV

- Improved model fit to market data (captures better market dynamics) thus it provides better fit to volatility dynamics. Many studies support this including Alfeus et al (2023).
- Enhanced pricing performance and why it matters to the practitioners.
- Risk Management: Financial institutions use rough volatility models to improve risk management practices by better accounting for extreme events and tail risk.

Conclusion

It's important to note that while rough volatility models have gained considerable attention for their ability to capture real-world market dynamics, they are not without their own complexities and challenges. Traders, investors, and risk managers need to carefully consider the implications of rough volatility in their decision-making processes and adapt their strategies accordingly.

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